Symmetry and Statistical Properties of Nuclei

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Abstract

Symmetries and statistical properties are strongly connected in nuclei. Statistical distributions can provide information about the underlying character of nuclei.

One of the clear examples of the relation between symmetry and statistical behavior is the large enhancements of parity violation in neutron resonances. The underlying character of nuclear properties could be provided from statistical distributions. It was found for electromagnetic transitions that the symmetry breaking changes the transition distribution from the standard Porter-Thomas distribution to another one. It was also found that a small symmetry breaking can have a large effect on the eign value distribution, and can change the statistical distribution of electromagnetic transitions. Electromagnetic transition distributions and nuclear level statistics have been used successfully to provide unique tests of predictions of random matrix theory.

Introduction

It is appropriate to examine the relation between symmetries and statistical concepts, since the nucleus is well described by statistical approaches such as Random Matrix Theory (RMT).

In addition to fundamental symmetries (isospin, K quantum number, L quantum number, etc) which are important to practicing nuclear physicists, the interplay between approximate symmetries and statistical properties is also interesting.

The fundamental symmetry that could be considered is parity as a classic example of the effect of statistical properties on physical observables. The size of the longitudinal symmetry (helicity dependence of the total cross section) will be discussed below. The neutron resonance in heavy nuclei is amplified by a factor of 10^6 relative to the longitudinal symmetry for nucleon-nucleon system.

The details of these experimental results are presented in a comprehensive review [1].

Isospin can be considered as an approximate symmetry. A series of measurements have been performed on the effect of isospin symmetry breaking on level statistics [2,3] and on electromagnetic transitions[4].

The most striking feature is that a small symmetry breaking can have a large impact on the level statistics and on the transition distributions.

Weak interaction in nuclei had always a dual role. The basic problem of many-particle theory is to understand and to choose the type of the effective weak interaction. Weak interaction can be used as a probe of this strongly interacting system.

In a similar manner one can use statistical concepts (RMT) to learn about nuclear properties, or use the nucleus as a laboratory to test explicit predictions of RMT.

Theory

Neutron Resonances Parity Violation

The weak force in nuclei has been studied through measurement of parity violation observables in nucleon-nucleon scattering, in a few body systems and in light nuclei. The strength of weak interaction is about 10^{-7} that of the strong force. Nucleon-nucleon scattering were summarized by Adelberger and Haxton [5].

The recent accepted explanation about parity violation in neutron-induced reaction was provided by Flambaum [6]. He estimated the size of the weak matrix element V_{sp}^{j} between s-and p-wave resonances to be 1 meV and predicated a longitudinal asymmetry to be:

$$p = \frac{\sigma_{+} - \sigma_{-}}{\sigma_{+} + \sigma_{-}} = 2\sum \frac{V_{sp}^{j}}{E_{s} - E_{p}} \sqrt{\Gamma_{ns} / \Gamma_{np}} \qquad \ldots (1)$$

Where:

σ_+ and σ_- are the + and – helicity cross section for the p-wave of compound nucleus (CN).

E_s-E_p is the close spacing between s and p waves

Γ_{ns}, Γ_{np} are gamma functions for neutron s and p waves

Σ is a summation over all possible states

From equation 1 the absorbed symmetry is a sum of terms of a product of the weak matrix element (V_{sp}^{j}) between two states times the constants representing the resonance energies and widths ($\Gamma_{ns} / \Gamma_{np}$).

The explanation for the large asymmetries observed is that very large S-wave resonance are mixed into very small p-wave resonances, producing what in called kinematical enhancement. The matrix elements between these very complicated states (for heavy nuclei there are of the order of 10^5 components in the CN wave functions) and the close spacing (E_s-E_p) lead to another enhancement usually called dynamical or statistical enhancement. The history and the development of this subfield are briefly reviewed by Michell et al [1].

An extended work was formed for parity violation measurements to higher neutron energies to more resonances, and to a wider range of targets [1].

<u>Violation Of Isospin</u>

a- *Effect on level statistics of isospin symmetry breaking*

The random matrix theory has been employed in a wide variety of applications [7], such as compounds nucleus states and in few experimental nuclear tests.

The main reason is that the standard measurements which are used to analyze the level statistics are sensitive to misasignment quantum numbers and to missing levels.

These stringent requirements (usually called purity and completeness) have limited the experimental tests. The role of symmetries in RMT is crucial.

Since the symmetries gover the statistical distributions, and there is much approximate symmetry in nuclei, it is natural to consider the effect of symmetry breaking on these distributions. The only direct way to test this is to find all of the states in an energy region and to determine all of their quantum numbers.

The RMT was used to measure the effect of isospin symmetry breaking, since isospin is considered to be very well understood. To maximize the symmetry breaking the nuclides ^{26}AL and ^{30}P have been studied.

The standard version of RMT that applies to nuclei is the Gaussian Orthogonal Ensemble (GOE).

The GOE prediction of the nearest neighbor spacing S is very close to the Wigner distribution [2]

$$P_{GOE}(x) = \frac{\pi\ x}{2} e^{-\pi\ x^2/4} \qquad \ldots (2)$$

Where $x = \frac{S}{D}$, with D is the average spacing for ^{26}AL and ^{30}P. N=Z=odd have the feature of T=0 and T=1 states coexist from the ground state.

b- *Effect of isospin symmetry breaking on Electromagnetic transitions*

A Gaussian distribution is predicted from the GOE. The observable is the square of the amplitude and the corresponding distribution in $x^2$of one degree of freedom, the Porter-Thomas (PT) distribution is [8]:

$$PT(y) = \frac{1}{\sqrt{2\pi y}} e^{\frac{-y}{2}} \qquad \ldots (3)$$

Where y is the dimensionless strength parameter.

For the study of electromagnetic transitions it is convenient to define y=B(XL)/< B(XL)>, where B is the reduced matrix, X is the type of electromagnetic transition and L is the multipolarity of the transition.

In addition the transitions are labeled by whether or not the isospin changes. The different types of transitions have very different average strength, and then the average matrix element must be determined separately for each B(X L ΔT). For z=log(y).

The PT distribution becomes:

$$PT(z) = \ln 10 \sqrt{y/2\pi}\; e^{-y/2} \qquad \ldots (4)$$

Results and Discussion

Fig.1 illustrates the parity violation in raw data by inspection for U^{238} transition of two helicity states near the 63.4 ev resonance.

There are longitudinal asymmetries (the largest measured value was 14%) but the dramatic evidence for the essential correctness of Flambaum theory [1] gave a small values only, and since the wave functions were too complicated, no quantities information could be obtained.

To solve this problem, one can use purely statistical approach; where the compound nucleus behaves statistically and the fluctuation properties of the compound nucleus states suggests that the weak matrix elements treated as random variables. From equation 1 the measured asymmetry is the sum of random variables (the weak matrix elements between the compound nucleus states) and is itself a random variable. All the weak matrix elements are sampled from the same distribution. It is diffeclt to determine the value of the individual weak matrix element, but it is possible to determine from the set of measured asymmetries the variance of distribution of the individual weak matrix elements. Therefore the experimental result is the root mean square (rms) of the weak matrix element for a given nuclide.

From these values it was possible to determine the effective weak nucleon-nucleus interaction.

Within experimental errors this effective weak interaction is constant as a function of mass number.

The statistical nature of the compound nucleus system led to a rather straightforward method of obtaining the rms weak matrix element. The parity violation neutron resonance experiments are the most striking example of the impact of the statistical nature of the system on a physical observable.

For the violatin of isospin and for the effect on level statistics equation 2 shows that if the isospin is a good quantum number, then a mixture of T=0 and T=1 states observed in the measurements should be a random mixture of two GOE_S. Since isospin is known to be broken at a few percent levels in these nuclei, one might expect only a small change from two GOE distributions. Experimentally the distribution is intermediate between the one of GOE and two GOE distributions [2,3]. This indicates that a small symmetry breaking can have a large effect.

The governing parameter is $\lambda - \frac{\alpha}{D}$ where α the size of the symmetry is breaking. This means that even a very small symmetry breaking can have a large effect in a sufficiently dense spectrum.

For the effect of isospin symmetry breacking on electromegnatic transitions the results of ^{26}AL and ^{30}P, although the strength of the conclusions is limited by the statistics, there are two rather striking results. The transition distributions do not agree with the PT distribution, and the distributions for the different types of transitions are not the same.

Although there were many data sets that agree with the PT distribution, there was no theoretical prediction for the effect of symmetry breaking on the distributions. Heuristic arguments strongly suggested that the transition distribution should not be change from Porter-Thomas in direct contradiction to the experimental results [8,9].

There is a new issue for the transition distributions, where the effect of symmetry breaking on the eigen-value distribution was a generic effect, which could be explained completely with RMT. There is a deviation between RMT and Porter-Thomas distribution. For example if E1 and M1 transitions are different, then this must be a dynamical effect which can not be explained by a theory such as RMT. Thus the approach of using a high statistics measurement to confirm the experimental results can not used. To simulate the dynamics of many body problem, shell model have been used to study the effect of isospin symmetry breaking on the different types of electromagnetic transitions. This approach is model dependent and not a straight forward as the RMT.

Conclusion

Statistical distributions can provide information about the underlying character of nuclear properties. The most striking result is the extremely large enhancement of parity violation in neutron resonance.

The general conclusion for the electromagnetic transition is that symmetry breaking changes the transition distribution from the standard Porter-Thomas distribution. This opens the possibility to invert this logic and to use measured distributions of electromagnetic transition to infer characteristics of the relevant approximate symmetries.

Thus the nucleus has proven to be a suitable laboratory to test predictions of statistical theory.

In particular the prediction that a small symmetry breaking can have a large effect on the eigen value distribution was demonstrated explicitly for the first time. In addition, it was demonstrated for the first time that symmetry breaking does change the statistical distribution of electromagnetic transitions.

These measurements preceded the theoretical analysis, which are consistent with the unexpected experimental results.

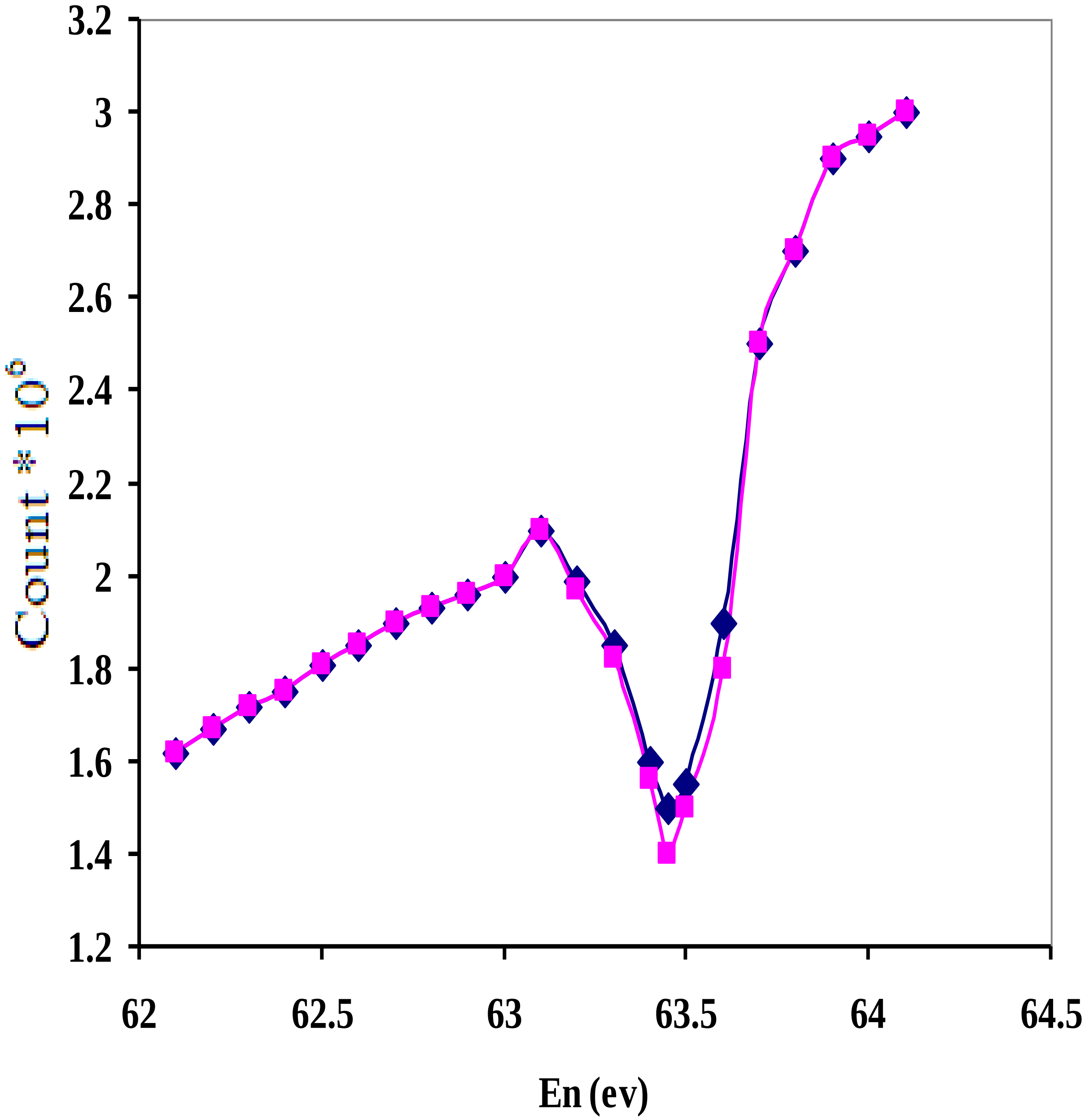

Figure 1: ^{238}U transmission spectra for two helicity near the 63.4 ev resonance

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الخواص التناظرية والاحصائية للنوى

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الخلاصة

خاصية التناظر والخواص الاحصائية للانوية مرتبطة مع بعضها بقوة. ان التوزيعات الاحصائية يمكن ان تعطينا معلومات على خواص غير ظاهرة للانوية.

احد الامثلة الواضحة على العلاقة بين التناظر والسلوك الاحصائي هو خرق التناظر في مواقع رنين النيوترونات.

ان الصفات الداخلية للانوية يمكن الحصول عليها من التوزيعات الاحصائية. لقد تم ايجاد ان خرق التناظر يؤدي الى تغيير الانتقالات الكهرومغناطيسية من انتقالات التوزيع القياسي لبورتر- توماس الى توزيعات اخرى. وقد تم ايجاد انه عند خرق تناظري بسيط يؤدي الى تاثير كبيرعلى توزيعات قيم ايجن.

ان توزيعات الانتقالات الكهرومغناطيسية و احصائية المستويات النووية تم استخدامها بنجاح في الحصول على اختبارات فريدة لتوقعات نظرية المصفوفات العشوائية.