On Feebly Dispersive G-Spaces

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Abstract: *In this paper we introduce a new type of G-spaces which we call it a feebly dispersive G-space. We give the definition by depending on the definition of an feebly neighborhood which itself depends on the concept of an feebly-open set . Also, we study its equivalent definitions, properties, subspace, product space, and equivariant homeomorphic image . Moreover we study the relation between the feebly dispersive G -spaces ,and each of the feebly Cartan G-spaces ,and the sets* $J^f(x)$ *and* $\Lambda^f(x)$ *respectively. Finally we give an example when the converse may not be true .*

Key words: feebly limit set of x , feebly prolongational limit set of x , feebly Cartan G-space, feebly dispersive G-space .

Introduction

The concept of semi-open sets was first introduced by Levin, N. in [1], while the concept of feebly-open sets was introduced by

Navalagi, G.B. in [2]. Recall that a subset A of a topological space (X,τ) is said to be semi-open (briefly s-open) set if there exists an open subset U of X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is defined to be semi-closed (briefly s-closed) . The intersection of all s-closed subsets of X containing a set A is called the semi-closure (briefly s-closure) of A, and it is denoted by $\overline{A}^s$. A subset A of a topological space (X,τ) is said to be feebly-open (briefly f-open) set if there exists an open subset U of X such that $U \subseteq A \subseteq \overline{U}^s$. The complement of an feebly-open set is defined to be feebly-closed (briefly f-closed) . The purpose of this paper is to introduce a new type of G-spaces which we call an feebly dispersive G-space. The characterizations and basic properties of feebly dispersive G-spaces have been studied. Moreover, the relation between the feebly dispersive G-spaces and each of the feebly Cartan G-spaces, and the sets $J^f(x)$ and $\Lambda^f(x)$ respectively was introduced. An example when the converse may not be true is given also.

1. Preliminaries:

First we recall the definitions and theorems which we need .

1.1 Definition [3]:

A topological transformation group is a triple (G,X,π), where G is a topological group, X is a topological space and $\pi: G \times X \to X$ is a function such that:-

- **i.** π is continuous.
- **ii.** $\pi(e,x) = x$, for each $x \in X$, where e is the identity element of G.
- **iii.** $\pi(g_1,\pi(g_2,x)) = \pi(g_1 g_2,x)$, for each $x \in X$ and $g_1, g_2 \in G$.

The function π is called an action of G on X.

1.2 Remarks:

- **i.** If there is no more than one action of G on X, we write gx instead of $\pi(g,x)$ and (G, X) instead of (G,X,π).
- **ii.** We mean by a G-space X a topological transformation group

(G,X) where X is a completely regular Hausdorff space and G is a locally compact non-compact toplological group .

1.3 Definition[4]:

If U and V are subsets of a G -space X , then U is said to be thin relative to V if the set $((U,V)) = \{g \in G : gU \cap V \neq \phi\}$ is relatively compact in G . If U is thin relative to itself, then it is called thin.

1.4 Definition[2]:

A subset A of a topological space X is said to be feebly neighborhood (f-neighborhood) of a point x in X if there exists an f-open set U in X such that $x \in U \subseteq A$.

1.5 Definition[5]:

A G -space X is called an feebly Cartan (written f-Cartan) G -space if every point of X has a thin f-neighborhood .

1.6 Definition[6]:

A subset A of a G -space X is said to be invariant under a subset S of G if $SA \subseteq A$, where $SA = \{sa : s \in S, a \in A\}$.

1.7 Definition[6]:

A subset A of a topological group G is said to be syndetic in G if there is a compact subset K of G such that $G = AK$.

1.8 Definition[3]:

Let X be a G -space and $x \in X$. Then:-

- **i.** $G_x = \{g \in G : gx = x\}$ is called the stability subgroup of G at x .
- **ii.** $Gx = \{gx : g \in G\}$ Is called the orbit of x under G .

1.9 Definition[3],[6]:

Let X be a G -space and $x \in X$. Then the point x is said to be:-

- **i.** Fixed point if $gx = x$, for each $g \in G$.
- **ii.** Periodic point if G_x is syndetic in G .

1.10 Definition[7]:

Let X be a G-space. A subset S of X with $S \neq X$ is said to be star if for each $x \in X$ there exists $g \in G$ such that $gx \in S$.

1.11 Definition[5]:

Let X be a G-space and $x \in X$, then:-

$J^f(x) = \{y \in X : \exists$ a net (g_α) in G and a net (x_α) in $X \ni g_\alpha \to \infty$ and $x_\alpha \xrightarrow{f} x$ such that $\pi(g_\alpha, x_\alpha) = g_\alpha x_\alpha \xrightarrow{f} y\}$.

$\Lambda^f(x) = \{y \in X : \exists$ a net (g_α) in $G \ni g_\alpha \to \infty$ and $\pi(g_\alpha, x) = g_\alpha x \xrightarrow{f} y\}$.

Where $J^f(x)$ and $\Lambda^f(x)$ are called the feebly prolongational limit set of x and the feebly limit set of x respectively. It is clear that $\Lambda^f(x) \subseteq J^f(x)$. The notation "$g_\alpha \to \infty$", means that (g_α) has no convergent subnet.

1.12 Theorem[5]:

Let X be a G-space and $x \in X$, then :

i. If $x \notin \Lambda^f(x)$ for each $x \in X$, then the stability subgroup of G at x is compact.

ii. $\overline{Gx}^f = Gx \bigcup \Lambda^f(x)$.

1.13 Definition[3]:

Let (G, X, π_1) and (G, Y, π_2) be topological transformation groups. A continuous function $\lambda : X \to Y$ is called an equivariant function if λ satisfies:

For each $g \in G$, $x \in X$, $\lambda(\pi_1(g, x)) = \pi_2(g, \lambda(x))$ or simply, $\lambda(gx) = g\lambda(x)$.

1.14 Definition[8]:

A function $f : X \to Y$ from a topological space X into a topological space Y is said to be feebly open (f-open) if the image of every open subset of X is an f-open set in Y.

2. Feebly Dispersive G-space.

In this section we introduce a new type of G-spaces, which we call a feebly dispersive G-space. This space is stronger than an feebly Cartan G-space. Besides we give examples and theorems.

2.1 Definition:

A G-space X is called an feebly dispersive (written f-dispersive) G-space if for each two points x and y of X, there are feebly neighborhoods U of x and V of y such that the set $((U,V)) = \{g \in G : gU \bigcap V \neq \phi\}$ is relatively compact in G .

2.2 Examples:

i. $(\Re,+)$ with the usual topology is a locally compact non-compact topological group. Also, $\Re$ with the usual topology is a completely regular Hausdorff space. Then $\Re$ acts on itself as follows:

$\pi : \Re \times \Re \to \Re$ which is defined by: $\pi(r_1, r_2) = r_1 + r_2$ for each $r_1, r_2 \in \Re$. It is clear that $\Re$ is an $\Re$-space. To prove that $\Re$ is an f-dispersive $\Re$- space. Let $x, y \in \Re$ and U, V be any two f-neighborhoods of x and y respectively where $U = (x - \in_\circ, x + \in_\circ)$ and $V = (y - \in_1, y + \in_1)$, $\in_\circ, \in_1 > 0$. Then the set:

$((U,V)) = \{r \in \Re : r + U \bigcap V \neq \phi\} = (y - x - (\in_\circ + \in_1), y - x + (\in_\circ + \in_1))$

is relatively compact in $\Re$. Thus $\Re$ is an f-dispersive $\Re$-space.

ii. $(\Re \setminus \{0\}, \cdot)$ with the usual topology is a locally compact non-compact topological group . Also, $\Re^2$ with the usual topology is a completely regular Hausdorff space. Then $\Re \setminus \{0\}$ acts on $\Re^2$ as follows :

$\pi : \Re \setminus \{0\} \times \Re^2 \to \Re^2$ is defined by :

$\pi(r, (x, y)) = (rx, ry)$ for each $r \in \Re \setminus \{0\}, (x, y) \in \Re^2$.

It is clear that $\Re^2$ is an $\Re\setminus\{0\}$-space. But $(0,0)\in\Re^2$ and every two f-neighborhoods U and V of $(0,0)$,the set $((U,V))=\Re\setminus\{0\}$ is not relatively compact in $\Re\setminus\{0\}$. Thus $\Re^2$ is not an f-dispersive $\Re\setminus\{0\}$-space.

2.3 Theorem:

Let X be a G-space. Then the following statements are equivalent:

i. X is an f-dispersive G-space.

ii. $J^f(x)=\phi$ for each $x\in X$.

iii. $y\notin J^f(x)$ for each $x,y\in X$.

Proof:

$(i)\to(ii)$.

Suppose that $J^f(x)\neq\phi \Rightarrow \exists\, y\in X$ such that $y\in J^f(x)$, then by definition (1.11) there is a net (g_α) in G and a net (x_α) in X $\ni g_\alpha\to\infty$ and $x_\alpha \xrightarrow{f} x$ such that $\pi(g_\alpha,x_\alpha)=g_\alpha x_\alpha \xrightarrow{f} y$. Since $x,y\in X$ and X is f-dispersive, then there are f-neighborhoods U of x and V of y such that the set $((U,V))$ is relatively compact in G. Since $x_\alpha \xrightarrow{f} x$ and $g_\alpha x_\alpha \xrightarrow{f} y$, then $\exists\alpha_\circ\in D$ such that $x_\alpha\in U$ and $g_\alpha x_\alpha\in V$ for each $\alpha\geq\alpha_\circ$, hence $g_\alpha x_\alpha\in g_\alpha U$. Therefore $g_\alpha\in((U,V))$. Since $((U,V))$ is relatively compact in G, then the net (g_α) has a cluster point $g\in G$. This is a contradiction, since $g_\alpha\to\infty$.

$(ii)\to(iii)$. Clear.

$(iii)\to(i)$.

To prove that X is an f-dispersive G-space. Suppose not, then there are two points x and y of X such that for each f-neighborhoods U of x and V of y the set $((U,V))$ is not relatively compact in G , thus there is a net (g_α) in G such that $g_\alpha\to\infty$. Since $g_\alpha U\cap V\neq\phi$ for each $\alpha\in D$, then there is $x_\alpha\in U$, such that $g_\alpha x_\alpha\in V$ for each $\alpha\in D$. Hence (x_α) and $(g_\alpha x_\alpha)$ are nets in X such that $x_\alpha \xrightarrow{f} x$ & $g_\alpha x_\alpha \xrightarrow{f} y \Rightarrow y\in J^f(x)$ this is a contradiction. Thus X is an f-dispersive G-space.

2.4 Corollary:

If X is an f-dispersive G-space. Then $\Lambda^f(x)=\phi$ for each $x\in X$.

Proof:

Since X is an f-dispersive G-space, then by theorem (2.3), $J^f(x)=\phi$ for each $x\in X$. Since $\Lambda^f(x)\subseteq J^f(x)$ for each $x\in X$, then $\Lambda^f(x)=\phi$, for each $x\in X$.

2.5 Remark:

The converse of corollary (2.4) may not be true. Consider the following example:-

Example: $(Q,+)$ with relative usual topology is a topological group (where Q is the set of all rational numbers). Then Q acts on itself as follows: $\pi: Q\times Q\to Q$ which is defined by: $\pi(g,x)=g+x$, $\forall g,x\in Q$. Clear that (Q,Q) is a topological transformation group.

To prove that $\Lambda^f(x)=\phi$, $\forall\ x\in Q$, Let $y\in\Lambda^f(x)$, then there is a net (g_α) in Q $\ni g_\alpha\to\infty$ and $g_\alpha x=g_\alpha+x\xrightarrow{f}y$. Since $g_\alpha x\xrightarrow{f}y$, then by Remark ((1.2.19) in [9]), $g_\alpha x\to y$. Since $g_\alpha\to\infty$, then $g_\alpha x=g_\alpha+x\to\infty$.

This is a contradiction, since $g_\alpha x\to y$. Thus, $\Lambda^f(x)=\phi$, $\forall x\in Q$. But Q is not f-dispersive Q-space, since Q is not locally compact topological group.

2.6 Theorem:

An f-dispersive G-space is f-Cartan .

Proof:

Let $x\in X$. Since X is f-dispersive, then there exist f-neighborhoods U and V of x in X such that the set $((U,V))$ is relatively compact in G . By Proposition ((1.1.22) & (1.1.24) in [9]) $U\cap V$ is an f-neighborhood of x in X. Since $((U\cap V,U\cap V))\subseteq((U,V))$ and $((U,V))$ is relatively compact in G, then so is $((U\cap V,U\cap V))$. Hence $U\cap V$ is a thin f-neighborhood of x in X . Thus X is an f-Cartan G-space.

2.7 Remark:

The converse of theorem (2.6) may not be true .
Consider the following example :-

Example: $(\Re,+)$ with the usual topology is a locally compact non-compact topological group. Also, $D=\{(x,y)\in\Re^2\setminus\{(0,0)\}: x\geq 0, y\geq 0\}$ with the relative usual topology is a completely regular Hausdorff space. Then $\Re$ acts on D as follows: $\pi:\Re\times D\to D$ is defined by:
$\pi(t,(x,y))=(xe^{-t},ye^{t})$, for each $t\in\Re$ and $(x,y)\in D$. It is clear that D is an $\Re$-space.

To show that D is an f-Cartan $\Re$-space. Let $(x,y)\in D$ and $U=(x-\in,x+\in)$ be an f-neighborhood of x in $L=\{x\in\Re: x\geq 0\}$ where $x>\in>0$ and $W=\{y\in\Re: y\geq 0\}$ be an f- neighborhood of y. By proposition ((1.1.42) in [9]) $U\times W$ is an f- neighborhood of (x,y) in D. Before we prove $((U\times W,U\times W))=((U,U))$ we need to show that W is invariant under $\Re$. $(\Re,+)$ with the usual topology is a locally compact non-compact topological group and W as a subspace of D with the relative usual topology is a completely regular Hausdorff space. Then $\Re$ acts on W as follows:
$\pi:\Re\times W\to W$ is defined by : $\pi(t,y)=ye^{t}$ for each $t\in\Re$ and $y\in W$. It is clear that W is an $\Re$-space. Since $\pi(\Re,W)=W$, then W is invariant under $\Re$. Now to prove that $((U\times W,U\times W))=((U,U))$. Since W is invariant under $\Re$, then:-

$$g\in((U,U))\leftrightarrow gU\cap U\neq\phi\leftrightarrow(gU\cap U)\times W\neq\phi$$
$$\leftrightarrow(gU\times W)\cap(U\times W)\neq\phi\leftrightarrow(gU\times gW)\cap(U\times W)\neq\phi$$
$$\leftrightarrow g(U\times W)\cap(U\times W)\neq\phi\ \leftrightarrow g\in((U\times W,U\times W))$$

Hence $((U\times W,U\times W))=((U,U))$

Now, to show that $((U,U))$ is relatively compact in $\Re$.
Since

$$e^{-t_1}(x-\in)=x+\in\Rightarrow t_1=\ln(\frac{x-\in}{x+\in})\ \ \&\ \ e^{-t_2}(x+\in)=x-\in\Rightarrow t_2=\ln(\frac{x+\in}{x-\in})$$

Then the set $((U,U)) = \{t \in \Re : tU \bigcap U \neq \phi\} = (t_1, t_2)$ is relatively compact in $\Re$. That is $U \times W$ is a thin f-neighborhood of (x, y) in D. Hence D is an f-Cartan $\Re$-space. But D is not f-dispersive $\Re$-space, Since $(x,0)$ and $(0, y)$ are points in D, but every f-neighborhoods U of $(x,0)$ and V of $(0, y)$ the set $((U,V)) = \{t \in \Re : tU \bigcap V \neq \phi\}$ is not relatively compact in $\Re$.

2.8 Theorem:

If X is an f-dispersive G-space, then:-

- **i.** Each orbit of X is f-closed in X.
- **ii.** Each stability subgroup of G at x is compact .

Proof:

i. Since X is an f-dispersive G-space, then by corollary (2.4), $\Lambda^f(x) = \phi$ for each $x \in X$. By theorem (1.12), $\overline{Gx}^f = Gx \bigcup \Lambda^f(x)$, $\forall x \in X$

$\Rightarrow \overline{Gx}^f = Gx$, $\forall x \in X$. Hence by proposition in [10] Gx is an f-closed set in X, $\forall$ x $\in$ X.

ii. Since X is an f-dispersive G-space, then by corollary (2.4), $\Lambda^f(x) = \phi$ for each x $\in$ X. Thus by theorem (1.12) the stability subgroup of G at x is compact.

2.9 Theorem:

If X is an f-dispersive G-space, then:-

- i. There is no fixed point.
- ii. There is no periodic point.

Proof:

i. Let $x \in X$ such that x is a fixed point. Since X is an f-dispersive G-space, then there exist f-neighborhoods U and V of x in X such that the set $((U,V))$ is relatively compact in G. Because x is a fixed point, then $gx = x$ for each $g \in G$. So $gU \bigcap V \neq \phi$ for each $g \in G$, that is $((U,V)) = G$. Since $((U,V))$ is relatively compact in G, then G is compact. But G is not compact, which leads to a contradiction. Hence X has no fixed point.

ii. Let $x \in X$ such that x is a periodic point, then by definition (1.9) no. (ii) G_x is a syndetic subgroup in G. That is there is a compact subset K of G such that $G = G_x K$. By theorem (2.8) G_x is compact in G for each $x \in X$. Thus G is compact, but that leads to a contradiction, since G is not compact. Hence X has no periodic point.

2.10 Theorem:

If X is an f-dispersive G-space, H is a closed subgroup of G and Y is an f-open subspace of X which is invariant under H, then Y is an f-dispersive H-space.

Proof:

By Remark ((1.24) in [6]) we get (H,Y) is a topological transformation group . Since Y is a subspace of X and X is a completely regular Hausdorff space, then so is Y. Since G is locally compact and H is a closed subgroup of G, then so is H . Hence Y is an H-space. To prove that Y is f-dispersive, Let $x, y \in Y$, then $x, y \in X$. Since X is f-dispersive, then there exist f-neighborhoods U and V of x and y respectively in X such that $((U,V))$ is relatively compact in G. Let $U_1 = U \cap Y$ & $V_1 = V \cap Y$.

Since Y is an f-open subspace of X, then by proposition ((1.13) in [5]) we have U_1 and V_1 to be f-neighborhoods of x and y respectively in Y. Since $((U_1,V_1)) \subseteq ((U,V))$ and because $((U,V))$ is relatively compact in G, then so is $((U_1,V_1))$. Since H is a closed subgroup of G, then $((U_1,V_1))$ is relatively compact in H. Thus Y is an f-dispersive H-space.

2.11 Corollary(I):

If X is an f-dispersive G-space and Y is an f-open subspace of X which is invariant under G , then Y is an f-dispersive G-space .

Proof: It is obvious.

2.12 Corollary(II):

If X is an f-dispersive G-space and H is a closed subgroup of G, then X is an f-dispersive H-space .

Proof: It is obvious.

2.13 Theorem:

Let $\lambda : X \to Y$ be an equivariant homeomorphism function from an f-dispersive G-space X into a space Y. Then Y is an f-dispersive G-space.

Proof:

We have (G,Y) is a topological transformation group. Since X is a completely regular Hausdorff space, and since λ is homeomorphism, then by [11] we get Y to be a completely regular Hausdorff space. Hence Y is a G-space. Now, to prove that Y is f-dispersive. Let $y_1, y_2 \in Y$. Since λ is onto, then there exists $x_1, x_2 \in X$ such that $\lambda(x_1) = y_1$ & $\lambda(x_2) = y_2$. Since X is f-dispersive and $x_1, x_2 \in X$, then there exist f-neighborhoods U and V of x_1 and x_2 respectively in X such that $((U,V))$ is relatively compact in G. Since λ is a homeomorphism, then by proposition ((1.1.52) in [9]) we have $\lambda(U)$ and $\lambda(V)$ to be f-neighborhoods of y_1 and y_2 respectively in Y. To prove that $((U,V)) = ((\lambda(U), \lambda(V)))$, Since λ is 1-1 and equivariant function, then:

$g \in ((U,V)) \leftrightarrow gU \cap V \neq \phi \leftrightarrow \lambda(gU \cap V) \neq \phi \leftrightarrow \lambda(gU) \cap \lambda(V) \neq \phi$

$\leftrightarrow g\lambda(U) \cap \lambda(V) \neq \phi \leftrightarrow g \in ((\lambda(U), \lambda(V)))$.Hence $((U,V)) = ((\lambda(U), \lambda(V)))$. Because $((U,V))$ is relatively compact in G, then so is $((\lambda(U), \lambda(V)))$. i.e. for each two points y_1 and y_2 of Y, there are f-neighborhoods $\lambda(U)$ of y_1 and $\lambda(V)$ of y_2 in Y such that $((\lambda(U), \lambda(V)))$ is relatively compact in G . Thus Y is an f-dispersive G-space .

2.14 Theorem:

Let X and Y be G-spaces. Then $X \times Y$ is an f-dispersive G-space if at least one of them is an f-dispersive G-space .

Proof:

Let X be an f-dispersive G-space. By definition ((1.2.7) in [12]) we have $X \times Y$ is a G-space. To prove that $X \times Y$ is f-dispersive.

Let $(x_1, y_1),(x_2, y_2) \in X \times Y \Rightarrow x_1, x_2 \in X$. Since X is f-dispersive, then there exist f-neighborhoods U and V of x_1 and x_2 respectively in X such that $((U,V))$ is relatively compact in G. By proposition ((1.1.42) in [9]) we get $U \times Y$ and $V \times Y$ to be f-neighborhoods of (x_1, y_1) and (x_2, y_2) respectively in $X \times Y$. Since $((U,V)) = ((U \times Y, V \times Y))$ and $((U,V))$ is relatively compact in G, then so is $((U \times Y, V \times Y))$. i.e. for each two points (x_1, y_1) and (x_2, y_2) of $X \times Y$, there are f-neighborhoods $U \times Y$ of (x_1, y_1) and $V \times Y$ of (x_2, y_2) in $X \times Y$ such that $((U \times Y, V \times Y))$ is relatively compact in G . Thus $X \times Y$ is an f-dispersive G-space .

2.15 Theorem:

If a G-space X has a star thin f-open set U, then X is an f-dispersive G-space.

Proof:

Let $x, y \in X$. Since U is star, then by definition (1.10) there are $g_1, g_2 \in G$ such that $g_1 x \in U$ and $g_2 y \in U$. Hence $x \in g_1^{-1}U$ and $y \in g_2^{-1}U$. Since $\pi_g : X \to X$ is a homeomorphism for each $g \in G$, then by proposition ((1.1.52) in [9]) we get $g_1^{-1}U$ and $g_2^{-1}U$ are f-neighborhoods of x and y respectively in X . Since U is thin, then by theorem in [4] we get $((g_1^{-1}U, g_2^{-1}U))$ is relatively compact in G . i.e. for each two points x and y of X , there are f-neighborhoods $g_1^{-1}U$ of x and $g_2^{-1}U$ of y such that the set $((g_1^{-1}U, g_2^{-1}U))$ is relatively compact in G . Thus X is an f-dispersive G-space .

2.16 Theorem:

If X is an f-dispersive G-space and $x \in X$, then $g \to gx$ is an f-open function of G onto Gx .

Proof:

Let U be an open subset of G. To prove that Ux is f-open in Gx. i.e. $(G-U)x$ is f-closed in Gx. Let $y \in \overline{(G-U)x}^{f}$, then by proposition ((1.2.14) in [9]) there is a net $(g_{\alpha}x)$ in $(G-U)x$ such that $g_{\alpha}x \xrightarrow{f} y$. Since X is f-dispersive, then there exists V be a thin f-neighborhood of y. Fixing α_0, then $(g_{\alpha}g_{\alpha_0}{}^{-1})(g_{\alpha_0}x) = g_{\alpha}x \in V \Rightarrow g_{\alpha}g_{\alpha_0}{}^{-1} \in ((V,V))$. Since $((V,V))$ is relatively compact in G, then by theorem ((17.4) in [11]) $(g_{\alpha}g_{\alpha_0}{}^{-1})$ has a cluster point say g. Hence by theorem ((11.5) in [11]) $(g_{\alpha}g_{\alpha_0}{}^{-1})$ has a subnet $(g_{\alpha_u}g_{\alpha_0}{}^{-1})$ which converges to g. i.e. $g_{\alpha_u}g_{\alpha_0}{}^{-1} \to g \Rightarrow g_{\alpha_u} \to gg_{\alpha_0}$ and by theorem ((11.8) in [11]) we get $g_{\alpha_u}x \to gg_{\alpha_0}x$. Since $g_{\alpha}x \xrightarrow{f} y$, then by Remark ((1.2.19) in [9]) $g_{\alpha}x \to y \Rightarrow g_{\alpha_u}x \to y$. Since X is T_2, then by theorem ((13.7) in [11]) we have $y = gg_{\alpha_0}x \in (G-U)x$. Hence, $\overline{(G-U)x}^{f} \subseteq (G-U)x$. But by definition in [10] we have $(G-U)x \subseteq \overline{(G-U)x}^{f}$. Therefore $(G-U)x = \overline{(G-U)x}^{f} \Rightarrow (G-U)x$ is f-closed in $Gx \Rightarrow Ux$ is f-open in Gx. Thus by definition (1.14) $g \to gx$ is an f-open function of G onto Gx.

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حول فضاءات ـ G المشتتة الضئيلة

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المستخلص:

في هذا البحث قدمنا نوعا جديدا من فضاءات–G أسميناه بفضاء-G المشتت الضئيل قدمنا التعريف اعتمادا على تعريف الجوار الضئيل الذي بدورة يعتمد على تعريف المجموعة المفتوحة الضئيلة. كذلك درسنا مكافئاته، خصائصه، فضائه الجزئي، جدائه وصورة التكافؤ التبولوجي المتساوي التغير. بالإضافة إلى ذلك درسنا العلاقة بين فضاءات-G المشتتة الضئيلة وكل من فضاءات-G لكارتان الضئيلة والمجموعات $J^f(x)$ و $\Lambda^f(x)$ على التوالي مع أعطاء مثال للاتجاه غير الصحيح.