# Nuclear Level Densities Calculations

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## Abstract

There is a need to determine nuclear level densities since experiments that defect nuclear level densities can not observe some of these densities, because of experimental limitations. The missed level could be calculated using standard correction of Porter-Thomas width distribution. A method that could determine the distance between levels distribution was developed. This method was applied to nuclear resonance data sets.

There was a good agreement between the calculated results and those from literatures of experimented data.

# Introduction

The study of nuclear structure needs information of nuclear level densities in many applications.

The shell model Monte Carlo calculation [1] recently gives method of calculating level densities and gives explicit predications of the parity and angular momentum dependence of level densities. It is important to consider the uncertainties in level densities observed from nuclear resonances because the average level densities are not suitable to answer all the questions about nuclear level densities.

In any experimental data set, some levels can be missed because of experimental limitation. The Porter Thomas distribution can be used as an approach to the correction for missing levels so that all levels below some cutoff are not observed. With this assumption and with experimental data distribution, one can estimate the missing levels. The Porter–Thomas distribution follows from Random Matrix Theory (RMT) [2]. Another prediction of RMT is that the nearest–neighbor spacing should obey the Wigner distribution. For RMT the widths and spacing of the levels are not correlated, so the analysis of the spacing distribution provides an independent determination of missing level fraction. A general expression for the spacing distribution have been developed when a fraction of the levels is missed, and a practical method have been also developed to analyze such data.

## Analysis Method For Spacing

The spacing between neighbored levels for a perfect sequence can be described by the Wigner distribution as follows:

$$P(X) = \frac{\pi}{2} X \; e^{-\pi \; X^2/4} \qquad \ldots (1)$$

Where: $X = \frac{S}{D}$

S is the spacing between neighbored levels, D is the average spacing when a level is missed. This means that two nearest spacing is included in the data set which should be not. The probability function density for an imperfect sequence must reflect the presence of higher order spacing.

The average spacing experimental data $D_{ob}$, differs from the true value D according to:

$$D = f\, D_{ob} \qquad \ldots (2)$$

where f is the fraction of levels observed experimentally.

A variable **z** can be defined as

$$z = \frac{S_{ob}}{D_{ob}} \qquad \ldots (3)$$

where $z = f\, X$ ... (4)

Spacing distribution can be written as:

$$P(z) = \sum_{k=0}^{\infty} a_k \lambda P(k; \lambda z) \qquad \ldots (5)$$

Where $a_k$ gives the relative contributions of the k-th nearest neighbor spacing distribution, λ is a parameter that characterizes the incompleteness of the sequence.

The average value of z=1 and the distribution P(z) has total probability of 1. The function P (k; λz) must correspond to total probability of 1 and an average value k + 1 in terms of the variable x. These relations give:

$$\sum_{k=0}^{\infty} a_k = 1 \qquad \text{... (6)}$$

$$\sum_{k=0}^{\infty} a_k (k+1) = \lambda \qquad \text{... (7)}$$

To determine $a_k$ we define S $\{a_k\}$ as;

$$S\{a_k\} = -\sum_{k=0}^{\infty} a_k \ln a_k \qquad \text{... (8)}$$

To calculate $\{a_k\}$, we define two multipliers α and β, maximizing S to these constraints α and β requires that;

$$\delta\left\{-\sum_{k=0}^{\infty} a_k \ln a_k - \alpha\sum_{k=0}^{\infty} a_k - \beta\sum_{k=0}^{\infty}(k+1)a_k\right\} = 0 \qquad \text{... (9)}$$

The result is $a_k = f(1-f)^k$ and $\lambda = \frac{1}{f}$.

Combining these results yields the final expression

$$P(X) = \sum_{K=0}^{\infty} f(1-f)^k P(k; X) \qquad \text{...(10)}$$

For $f$ = 1 this reduces to the Wigner distribution P (0; X).

The functions P (1; X) and P (2; X) can be calculated numerically; while for k ≥ 3 the kth order spacing distribution is well approximated by a Gaussian distribution centered at k + 1.

## Numerical Data Analysis

The likelihood function is

$$L = \Pi_i P(X_i) \quad \text{... (11)}$$

where the product is over all spacing's in the sequence. It is convenient to evaluate lnL. The most likely value of f is the one that maximizes equation (10) and the uncertainty in f is the deviation from the most likely value of f when lnL has decreased by 0.5 from its maximum value.

To test the method, a spectrum was generated and then levels were randomly removed to simulate the spectrum with f = 0.85. The corresponding spacing distribution is shown in Fig 1 upper. The normalized plot of lnL is shown in Fig 1 lower. The value of f was determined by the maximum likelihood analysis is 0.848 ± 0.018, which agree well with the true value of which is 0.85.

## Experimental Data Analysis

A collocation of proton resonance data sets was analyzed and it was found that the method worked very well. A data from n +$u^{238}$ reaction were considered, and we analyzed 237 S-wave resonance identified by Olsen et.al [3,4].

The reduced widths are shown in Fig 2 and compared with a truncated Porter-Thomas distribution [5].

The width analysis yields f =0.97 (+0.03, -0.08). The analysis indicates that essentially all of the resonances are observed. The nearest neighbor spacing distribution is shown in Fig 3. Our spacing analysis yields f = 0.89± 0.06. The spacing value of f seems the better estimate.

## Conclusion

Level densities are important for a variety of pure and applied nuclear physics. Resonances correspond to highly excited states that behave statistically are used in making correction for missing levels. In practice this correction has always been performed using the width distribution. The spacing analysis is more complicated, since the levels are missed at random.

The observed distribution have been assumed to has a weighted sum of probability distribution that two levels have no levels between them - P(0; x), P(1; x) -. A general expression has been derived to the imperfect

spacing distribution. After extensive testing with numerically generated data, the method was applied to proton and neutron resonances. In most cases the two analysis methods disagree, it is best to consider the data on a case by case basis. The spacing analysis method is much less sensitive to non statistical phenomena than is the width analysis method.

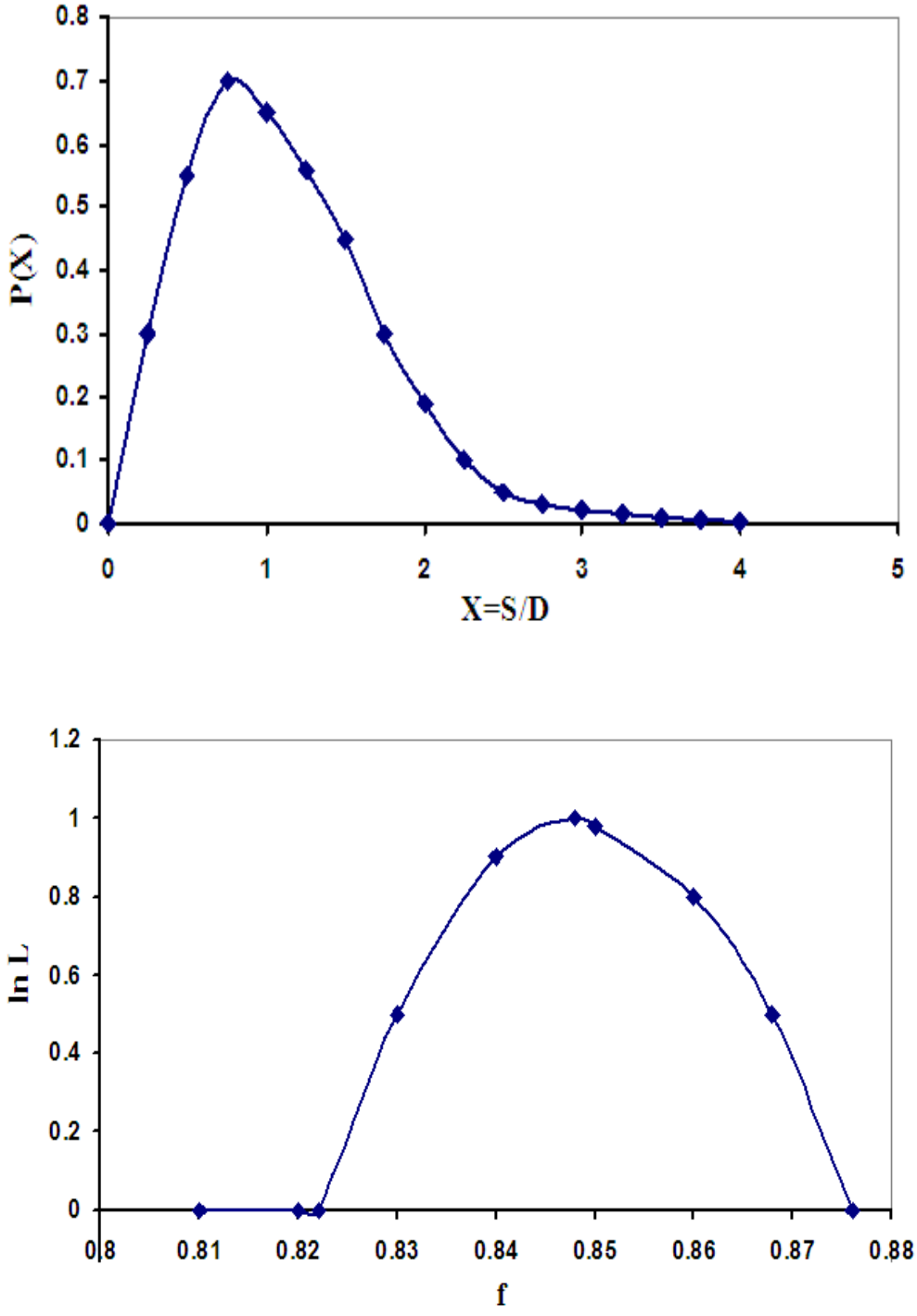



**Figure 1** The spacing distribution

Upper- For an imperfect Gaussian distribution' f=0.85.

Lower- The likelehood functions with a maximum at 0.848.

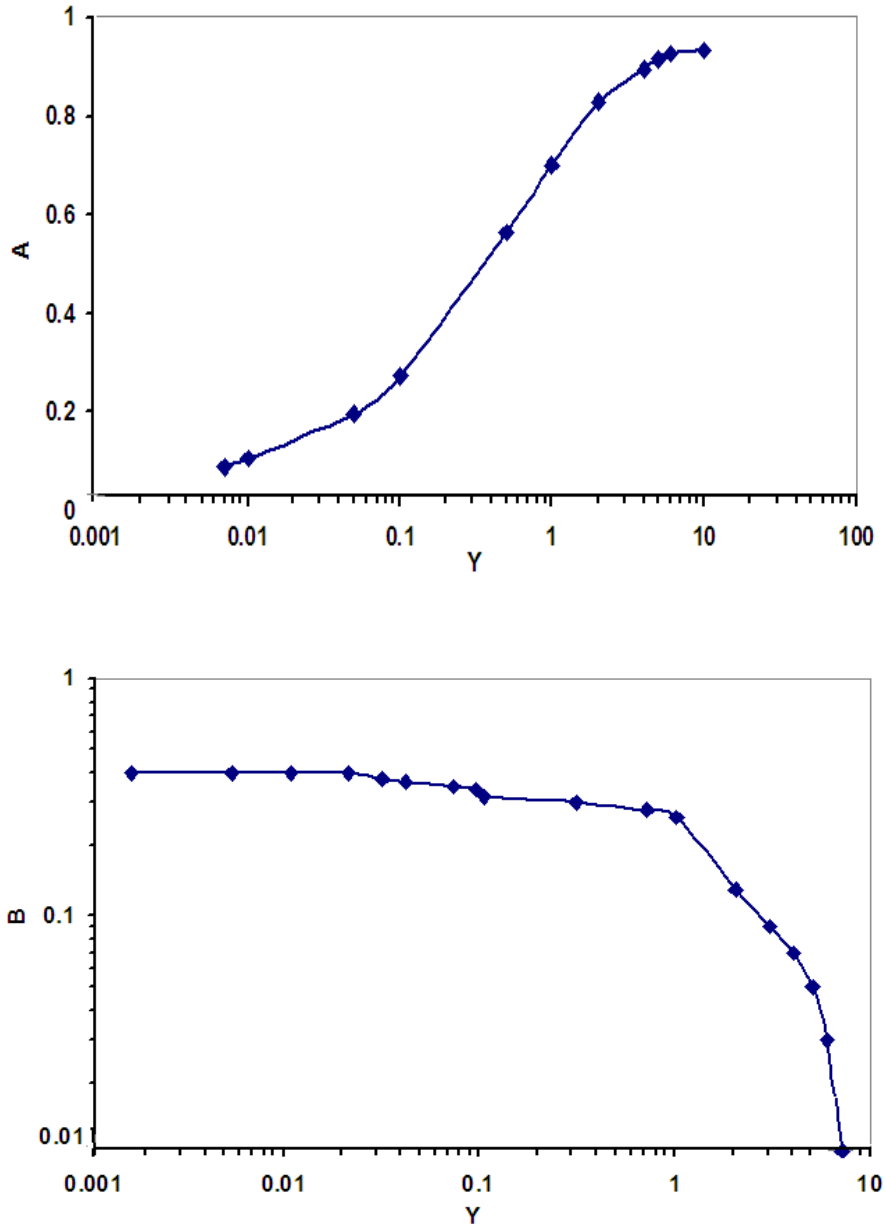
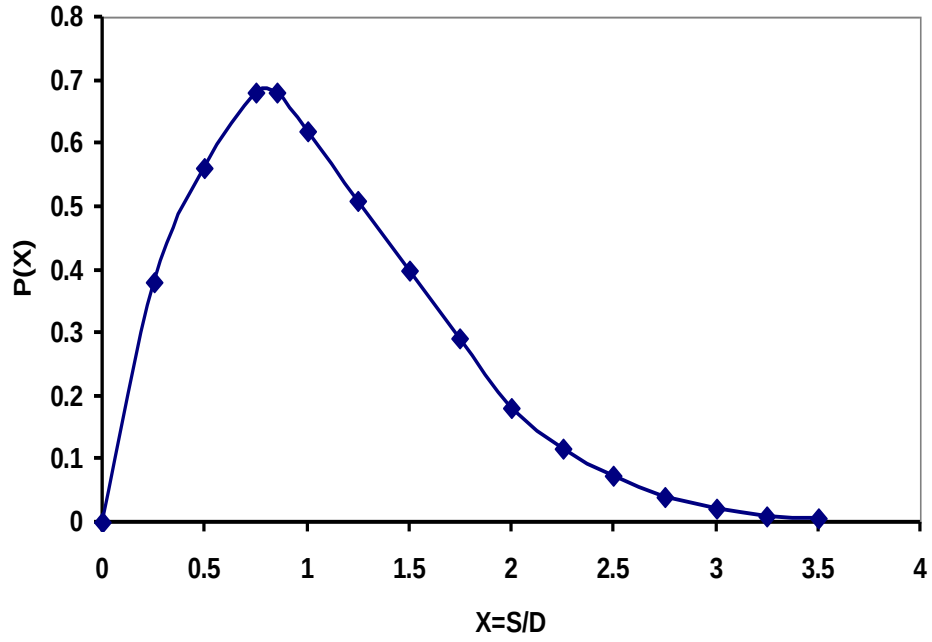



**Figure 2** Reduced width distributions for (n+u$^{238}$) S-wave resonances

Upper- Cumulative probability

Lower- The probability density function

(where Y= $\gamma^2/\langle\gamma^2\rangle$ , A= $\int P(Y)dY$ , B= $Y^{\frac{1}{2}}P(Y)$ )



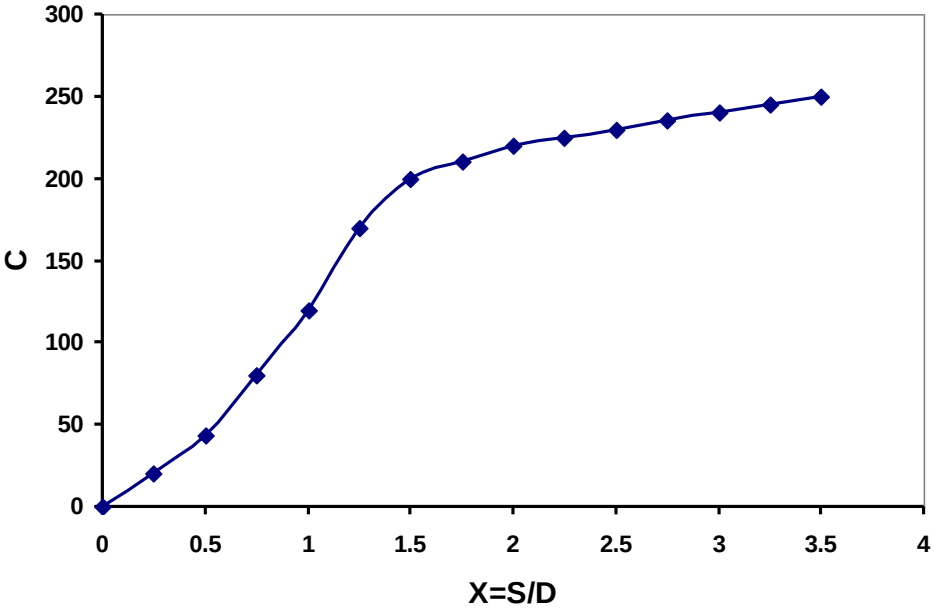




**Figure 3** a- Width distribution for (n+$u^{238}$) S-wave resonances

b- Cumulative sum of P(X)

(where C= $\int P(X)dX$ )

# References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# حساب كثافة المستويات النووية

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## الخلاصة

هنالك حاجة لايجاد كثافة المستويات النووية وذلك لان التجارب العملية المجراة للحصول على كثافة المستويات النووية لا يمكنها ملاحظة كثافة قسم من المستويات النووية. ان سبب ذلك يعود الى محددات في التجارب العملية. ان المستويات المفقودة في التجارب العملية يمكن اجراء حسابات لها باستخدام تصحيحات قياسية لتوزيعات السمك ل بورتر-تومسون.

تم تطوير طريقة باستطاعتها ايجاد المسافة بين المستويات الموزعة، هذه الطريقة يمكن تطبيقها على مجاميع من المعلومات الخاصة بالرنين النووي.

تم الحصول على تطابق جيد بين النتائج المستحصلة في حساباتنا ونتائج التجارب العملية الموثقة.